Construction of Character Table for C_{3v} Point group.

(i) There ar six symmetry operations present in C_{3v} point group i.e $E, C_3^1, C_3^2, \sigma_a, \sigma_b, \sigma_c$.
these operations are derive in the three classes i.e ($E, 2C_3, 3\sigma_v$) and hence there are three irreducible representations.
Let they are Γ_1, Γ_2 & Γ_3.

(ii) The sum of the squares of dimensions (i.e character of the identity operation) should be equal to six (or 6).

$$\sum l_i^2 = l_1^2 + l_2^2 + l_3^2 = 6$$

The values of l_i that will satisfy this requirement are 1, 1, and 2.

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1		
Γ_2	1		
Γ_3	2		

(iii) Every point group posesses one representation which is totally symmetric.
In this representations, all the operations have the character value one (1)
Thus we have

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1	1	1

It can be seen that the summation of the square, of the character of the operations is equal to 6

$$1^2 + 2 \times 1^2 + 3 \times 1^2 = 6$$

(iv) Γ_2 (Second irreducible representation), must be orthogonal to Γ_1.
since $\chi_2(E)$ must always be positive and hence Γ_2 must consist of three +1 and three −1. This is only possible if Γ_2 has 1, 1, and −1.

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1	1	1
Γ_2	1	1	−1

(v) One third representations will be of two and hence $\chi_3(E)$ is 2.

In order to find out the values of $\chi_3(C_3)$ and
we make the use of the orthogonality relationship.

Γ_1 and Γ_3 are orthogonal to each other
Γ_2 and Γ_3 are also

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1	1	1
Γ_2	1	1	-1
Γ_3	2	$\chi_3(C_3)$	$\chi_3(\sigma_v)$

$$\sum_R \chi_1(R)\cdot\chi_3(R) = (1)\cdot(2) + 2(1)\cdot\chi_3(C_3) + 3\cdot(1)$$

and $$\sum_R \chi_2(R)\cdot\chi_3(R) = (1)\cdot(2) + 2(1)\cdot\chi_3(C_3) + 3\cdot(-1)$$

Solving equation (i) and (ii) we get

$$2\chi_3(C_3) + 3\chi_3(\sigma_v) = -2$$

$$2\chi_3(C_3) - 3\chi_3(\sigma_v) = -2$$

On adding eqn (iii) and (iv) we get

$$4\chi_3(C_3) = -4$$

$$\therefore \chi_3(C_3) = -1$$

This value is substituted in equation (iii) to

$$2(-1) + 3\chi_3(\sigma_v) = -2$$

$$3\chi_3(\sigma_v) = 0$$

$$\therefore \chi_3(\sigma_v) = 0$$

Thus, the complete set of character of irreducible of C_{3v} Point group is:

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1	1	1
Γ_2	1	1	-1
Γ_3	2	-1	0

The complete character table for the point group C_{3v} is given as follows.

C_{3v}	E	$2C_3$	$3\sigma_v$		
A_1	1	1	1	z	$x^2 y^2, z^2$
A_2	1	1	-1	R_z	
E	2	-1	0	(x, y) (R_x, R_y)	(x^2-y^2, xy) (xz, yz)
I		II		III	IV

In the upper left corner is the Schoenflies notation for the group and the upper row of the table consist of the symmetry operations grouped in classes.

Area I represents the symbols for irreducible representations according to Mulliken.

Because first two irreducible representations are unidimensional and hence A or B may be used.

The character of principal axis of rotation for both the representations are symmetrical and hence A is used.

Subscript 1 is written for the symmetrical character (+1) of operation σ_v, while subscript 2 is written for unsymmetrical character (-1) of operation σ_v.

The symbol E shows the two dimensional representation.

Area II :- In area II of the table are the character of the irreducible representation of the point groups.

Area III :- Area III gives the transformation properties of Cartesian co-ordinates x, y, z and rotation about x, y & z axes i.e R_x, R_y and R_z.

Area IV :-
In this area IV of the table, squares and binary products of co-ordinates according to their transformation properties are described.

The squares of the vectors (x^2-y^2) and z^2 belong to A_1, x^2-y^2 and xy taken together and xz, yz taken together belong to E.

N_3